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NATURAL VIBRATIONS OF PIECE HOMOGENEOUS SPHERICAL BODIES TAKING INTO ACCOUNT THE WAVE DEPOSIT OF ENERGY

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Annotation
The natural vibrations of piecewise homogeneous spherical bodies located in an infinitely elastic medium are considered. A specker of complex mechanical systems is obtained, which is located in an elastic medium depending on various geometric and parametric parameters of the system. If we consider a rigid sphere in an infinitesimal elastic medium, rather than in elastic space, then it is shown what form the partial differential equation takes.

Key words:
Energy, spherical cavity, vibration, cavity, waves, stress, deformation medium, shear, frequency, elastic, mechanical systems

Let us consider the natural vibrations of piecewise homogeneous spherical bodies (Fig. 1) or in a spherical cavity located in an infinitely elastic medium.

Spherical cavity. In this case, there is no wave inside the sphere, so

$$\begin{bmatrix} \phi^{(\rho)} \\ \psi^{(p)} \end{bmatrix} = \begin{bmatrix} \phi \\ \psi \end{bmatrix} = 0 \quad \phi^{(\rho)} \text{ и } \psi^{\rho} = 0. \qquad (1)$$

Therefore, only two groups of coefficients need to be determined. $\{A_n\}$ и $\{B_n\}$

$$R = A_n J_n(\mu r) + B_n h_n(\mu r) \qquad (2)$$

Only two boundary conditions are necessary. We require that the voltage components be continuous at the boundary at $r = \alpha$

$$\sigma_{rr} = \sigma_{r\theta} = 0$$

This condition leads to the following equations for ω .

$$\xi_{11}^{(0)} \xi_{12}^{0} - \xi_{13}^{(0)} \xi_{14}^{(0)} = 0 \qquad (3)$$

Where

$$\xi_{11}^{(0)} = [\beta_1^2 - 2n(n-1)]h_n(\alpha_1) - 4\alpha_1 h_{n+1}(\alpha_1)$$

$$\xi_{12}^{(0)} = \left[\frac{1}{2}\beta_1^2 - (n^2 - 1)\right] h_n \psi_1 - \beta_1 h_{n+1}(\beta_1)$$

$$\xi_{13}^{(0)} = [2n(n+1)(n-1)]h_n(\beta_1) - \beta_1 h_{n+1}(\beta_1)$$

$$\xi_{14}^{(0)} = [(n-1)]h_n(\alpha_1) - \alpha_1 h_{n+1}(\alpha_1)$$

$$\alpha_1 = k_1 q = \frac{\omega^2}{c_{\rho 1}^2}\, q; \quad \beta_r = \frac{\omega^2}{c_{S1}^2}$$

The frequency equation is also solved by the Muller method on a computer. The asymmetric natural oscillations of a spherical hole are shown in Fig-2

In an incompressible medium $(c_\rho \to \infty$ and $\nu_1 = .0,5)$ attenuation, of course, is absent. The results of numerical calculations on a computer are presented in the table 1. by $n = 1$.

б) hard inclusions. If, instead of an elastic cavity, a rigid sphere is considered in an infinitely elastic medium, then the partial equation (3.) takes the form

$$\xi_{11}^{(1)}\xi_{12}^{(1)} - \xi_{13}^{(1)}\xi_{14}^{(1)} = 0 \qquad (4.)$$

Expressions $\xi_{11}^{(1)}$, $\xi_{12}^{(1)}$, $\xi_{13}^{(1)}$, $\xi_{14}^{(1)}$ they have the following form

$$\xi_{11}^{(1)} = (1 - \eta)h_1(\alpha_1) - \alpha_1\, h_2(\alpha_1)$$
$$\xi_{12}^{(1)} = 2(1 - \eta)]h_1(\beta_1) - \beta_1\, h_2(\beta_1)$$
$$\xi_{13}^{(1)} = 2(1 - \eta)]h_1(\beta_1)$$
$$\xi_{14}^{(1)} = (1 - \eta)]h_1(\alpha_1)$$
$$\eta = \rho_{c\rho}/\rho_{\text{вкл}}$$

The numerical results are shown in Table 2. When $\nu_1 = 0,25$.

As we can see, when $\eta \geq 1$ the real parts of the first natural frequency vanish.

Dependence of the complex natural frequencies of a spherical cavity on ν_1 (ν_1 environmental Poisson's ratio) .

Table -1

Ω	$\nu_1 = 0,25$	$\nu_1 = 0,35$	$\nu_1 = 0,40$	$\nu_1 = 0,45$	$\nu_1 = 0,5$
Ω	0.6019D+01 -0,7981D+ +00i	0,7501D+01 -0,6414D+ +01i	0,8541D+00 -0,5591D+01	0,9120D+00 0,4910D+00	0,7201D+00 0,0000D+00

The frequency equation of radial oscillation of a spherical elastic body is solved as a test problem. The results were compared with the results of the work [1].

By $\Phi(r, \theta, \Phi) = 0$, $\Psi = \theta = 0$ he frequency equation (3.) of a spherical body takes the form.

$(\lambda + 2 \cdot \mu)[(2 - h^2 a^2)\sinh a - 2 \cdot h \cdot a \cdot \cosh a] + 2\lambda(h \cdot a \cosh a - \sinh a) = 0$ (5)

where $h = \omega/c_\rho$. The numerical values of the solution of the frequency equation (5.) are given in Table 3.

As you can see from the table, the numerical results match after the decimal point to the fifth digit. Let us consider the natural vibrations of spherical inclusions located in an elastic medium. Dependence of complex frequencies of rigid spherical attraction on η

Table-2

η	Ω_n	$\Omega_I i$
0.2	0.253 D +00	0.4710 D -02
0.5	0.520 D +00	0.6148 D -02
0.8	0.8101 D +00	0.9118 D -02
1.0	0.1924 D -13	0.1211 D -11

Comparison of natural frequencies of an elastic medium.

Table-3

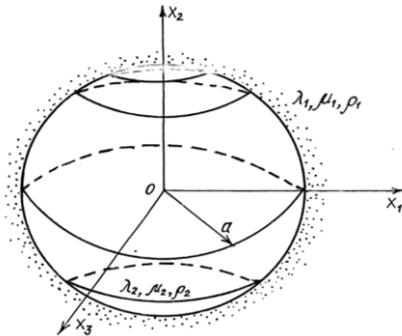| h	Our results	The results of the work [2]
0.8160	0.816025 D +00	0.816027D +00
1.9285	0.192846D +01	0.192843D +00
2.9359	0.293816D +01	0.293812D +00
3.9658	0.396472D +01	0.396478D+-00

Fig. 1
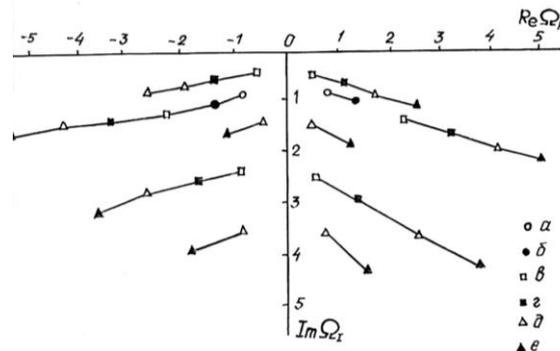Calculation scheme for spherical bodies located in an infinite environment.

Fig. 2
Distribution of the roots of the frequency equations

Torsional oscillations. They are characterized by the vanishing of the radial component of the mixing vector u, as well as dilation $d\iota\vartheta$ u. It is easy to see that in the general solution, they correspond to the part that includes the coefficients C_{mn}

$$\begin{pmatrix} \phi^{(\rho)} \\ \psi^{(p)} \end{pmatrix} = \begin{bmatrix} \phi^{(\rho)} \\ \psi^{(p)} \end{bmatrix} = \begin{bmatrix} \frac{\phi_0}{\Psi_0} \end{bmatrix} \sum_{n=0}^{\infty} E_n i^n \begin{bmatrix} J_n(ar) \\ J_n(\beta r) \end{bmatrix} \begin{bmatrix} \cos n\theta \\ \sin\theta \end{bmatrix} e^{-iwt}$$

Substituting this part into the boundary conditions leads to the following system of equations for determining the coefficients

C_{mn}^i и C_{mn}^e

$C_{mn}^i j_n(k_s^{(i)}R) = C_{mn}^{(e)} h_n(k_s^{(e)}R)$

$\mu_i C_{mn}^{(i)}[(n-1) j_n(k_s^{(i)}R) - (k_s^{(i)}R)j_{n+1}(k_s^{(i)}R)] =$
$= \mu_i C_{mn}^{(i)}[(n-1) h_n(k_s^{(e)}R) - (k_s^{(i)}R)j_{n+1}(k_s^{(i)}R)]$

From equating the determinant of the system to zero, we obtain the equation for the natural frequencies of torsional vibrations of a spherical inclusion:

$[n-1-g_i(\beta X)] = \rho[n-1-g_e(X)]$ (5)
Where
$g_i(z) = z j_{n+1}(z)/j_n(z)$, $g_e(z) = z j h_{n+1}(z)/j h_n(z)$,   X= $\omega\mathbb{R}/c_{se}$,

The dimensionless frequency reduced to the transverse velocity in the medium,

$\beta = c_{se}/c_{si}$

The ratio of transverse velocities outside (c_{se}) And inside (c_{si}) spherical inclusion, $\eta = \rho_e/\rho_i$ - the ratio of densities and $\rho = \mu_e/\mu_i = \beta^2\eta$ - the ratio of the shear modulus of the host medium and inclusion.

It is easy to see that equation (5) has a set of complex frequencies as its solution. $\chi^{(\kappa)} = \chi_0^{(\kappa)} + i\chi_0^{(\kappa)}$. The actual part $\chi_0^{(\kappa)}$ defines the natural frequency, and the imaginary part $\chi_i^{(\kappa)}$.- the corresponding attenuation coefficient. From a physical point of view, attenuation in an ideal elastic medium is explained by the radiation of energy from excited natural vibrations due to diverging spherical elastic waves. If we make a limiting transition in (5) corresponding to the case of the absence of a broadcasting medium (an isolated elastic ball at $\rho > 0$, then we will naturally arrive at the real frequency equation of torsional vibrations of the ball.

$n-1-g_i(\chi_{si})=0$ (6)

where $\chi_{si} = \omega\mathbb{R}/c_{si}$, -this equation defines a discrete spectrum $\chi_{si}^{(\kappa)}$ already valid frequencies, since there is no radiation. In the opposite case of a spherical cavity in a continuous elastic medium, when $\rho\to\infty$, we arrive at the complex frequency equation

$n-1-g_e(\chi)=0$ (7)

describing the spectrum of complex values $\chi^{(\kappa)}$ natural frequencies to the torsional vibrations of the cavity.

Let us expand (5) for an example in the case of n=1 U n=2. We get when n=1

$\beta_\chi \, ctg\beta_\chi = 1 - \dfrac{\beta^2\chi^2(1+i\chi)}{\chi^2\rho + 3(1-\rho)(1+i\chi)}$ (8)

And when n=2

$\beta_\chi \, ctg\beta_\chi = 1 - \dfrac{1}{3}\beta^2\chi^2 x$

$X\left\{1 + \dfrac{\beta^2\chi^2[\chi^{-i}(1-\frac{1}{3}\chi^2)}{-\frac{1}{3}\rho\chi^4 + [\chi^2(\rho-\beta^2) - 12(\rho-1)][\chi^{-i}(1-\frac{1}{3}\chi^2)]}\right\}$ (9)

Accordingly, these complex transcendental equations transform at $\rho \to 0$ в real equations for the natural oscillation frequencies of a ball, $\rho \to \infty - $ в into complex equations for natural frequencies and natural attenuation of torsional vibrations of the plane. It is when n=1

$\chi_{si}ctg\chi_{si} = 1 - \dfrac{\chi_{si}^2}{3}$ (10)

$\chi^2 - 3i_\chi - 3 = 0$ (11)

and at n=2

$\chi_{si}ctg\chi_{si} = 1 + \dfrac{4\chi_{si}^2}{\chi_{si}^2 - 12}$ (12)

$\chi^2 - 5i\chi^2 - 12\chi + 12i = 0$ (13)

It is interesting to note that equations (10) and (12), as well as the more general transcendental equations, include both trigonometric and algebraic functions. Due to the periodicity property of trigonometric functions, for each number n we will have an infinitely countable set of natural frequencies .The exception is the case of a cavity, when the natural frequencies are determined by algebraic equations of finite order, increasing with the number n. Spheroidal oscillations. This class of vibrations is characterized by the collapse to zero of the radial component of the rotation u in the general solution (3), this class corresponds to the part including the coefficients A_mp and B_mp. Substituting this part into the boundary

conditions (10) gives a homogeneous system of equations for determining the coefficients $A_{mn}^{(i)}$, $A_{mn}^{(e)}$, $B_{mn}^{(i)}$ и $B_{mn}^{(l)}$. As usual, the equality of the determinant of the system to zero means that it is compatible and leads to a transcendental equation for the natural frequencies of spheroidal oscillations. The determinant has the form

$$\Delta_n = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{vmatrix}$$

Where are the elements d_{ix} are expressed as follows:

$d_{11}=n-g_i(\alpha\gamma\chi)$, $d_{13}=n-g_e(\gamma\chi)$,

$d_{12}=n(n+1)$ $d_{14}=n(n+1)$,

$d_{21}=1$ $d_{23}=1$,

$d_{22}=n+1-g_i(\beta\chi)$, $d_{24}=n+1-g_e(\chi)$,

$d_{31} = n^2-n-\frac{1}{2}\beta^2\chi^2+2g_i(\alpha\gamma\chi)$, $d_{33} = n^2-n-\frac{1}{2}\chi^2+2g_e(\gamma\chi)$,

$d_{32}=n(n+1)[n-1-g_i(\beta\chi)]$, $d_{34}=n(n+1)[n-1-g_e(\chi)]$,

$d_{41}=n-1 - g_i(\alpha\gamma\chi)$,

$d_{43}=n-1 - g_e(\gamma\chi)$,

$d_{42} = n^2-1 - \frac{1}{2}\beta^2\chi^2+g_i(\beta\chi)$ $d_{44} = n^2-1 - \frac{1}{2}\chi^2+g_e(\chi)$,

Here $\alpha=\frac{c_{\rho e}}{c_{\rho i}}$- the ratio of the longitudinal velocities outside and inside the sphere, $\gamma=\frac{c_{se}}{c_{\rho e}}$- the ratio of the transverse and longitudinal velocities for the host medium, and the other designations have the same meaning as when considering torsional vibrations.

The transcendental equation for determining natural frequencies $\Delta_n=0$ (14)

It has a hard-to-see appearance in this case. At p→0, it turns into a real equation of proper spheroidal vibrations of the ball

$$\begin{vmatrix} n^2-n-\frac{1}{2}\chi_{si}^2+2g_i(\gamma_i\chi_{si}) & n(n-1)[n-1-(g_i\chi_{si})] \\ n-1-g_i(\gamma_i\chi_{si}) & n^2-1-\chi_{si}^2+g_i(\chi_{si}) \end{vmatrix} = 0, \qquad (15)$$

где

$$\gamma_i=\frac{c_{\rho e}}{c_{\rho i}}$$

By $\rho \to \infty$ − We arrive at a complex transcendental equation for the complex natural frequencies of spheroidal cavity vibrations:

$$\begin{vmatrix} n^2-n-\frac{1}{2}\chi^2+2g_e(\gamma\chi) & n(n+1)[n-1-g(\chi)] \\ n-1-g_e(\gamma\chi) & n^2-1-\chi^2+g_e(\chi) \end{vmatrix} =0, \qquad (16)$$

An important special case of spheroidal oscillations is radial oscillations. As follows from (9), at n=0, only the radial component of the displacement is nonzero. Movements occur only in the radial direction

$u_r = -A_{00}b_1(k_\rho r)$,

$\sigma_{rr} = \frac{\mu}{r}A_{00}[4b_1(k_p r)-[\frac{k_s}{k_p}] (k_s r) b_0(k_p r)]$

Using boundary conditions $u_r^{(i)} = u_r^{(e)}$ и $\sigma_{rr}^{(i)} = \sigma_{rr}^{(e)}$ by rR, as a result, we come to the following equation for the natural frequencies of radial oscillations of a spherical inclusion:

$$\alpha\gamma\chi\,ctg\,\alpha\gamma = 1 - \frac{\chi^2}{\eta}\,\frac{1+\iota\gamma\chi}{x^2+4\left(\frac{1}{p}-1\right)+i\gamma x} \qquad (17)$$

Similarly, there are two limiting cases here: radial vibrations of the ball ($\rho \to 0$) and cavities ($\rho \to \infty$). Accordingly, we get

$$x_{pi}\,ctg\,x_{pi} = 1 - \frac{x_{pi}^2}{4\gamma_i^2} \qquad (18)$$

$$X^2 - 4i\gamma x - 4 = 0. \qquad (19)$$

Where

$$x_{pi} = \omega R/C_{pi}$$

Analytical investigation of the equation (17) depending on the parameters $\alpha, \gamma, \eta,$ и ρ в in the general case, it is difficult. Nevertheless, it is possible to view the conditions under which the real and imaginary parts of the natural frequencies are small, i.e.

$$|x_{pi}| = |\alpha\gamma x| < 1$$

Decomposing $ctg \propto \gamma x$ on the left side (17) up to cubic terms, we arrive at a quadratic equation with respect to x_pi

$$\left[\frac{1}{a^2} - \frac{4}{15}\alpha\right]x_{pi}^2 - \frac{1}{a}\left[4a + \frac{3}{\eta\alpha^2}\right]X_p - \left[4a + \frac{3}{\eta\alpha^2}\right] = 0$$

Where

$\alpha = (\mu_e - \mu_i)/(\lambda_e + 2\mu_e)$. Note that

$4a + \frac{3}{\eta\alpha^2} = (4\mu_e + 3\lambda_\xi + 2\mu_1)/(\lambda_e + 2\mu_e) > 0.$

Condition $|X_{pi}| < 1$ it will be executed if

$$a\left[4a + \frac{3}{\eta\alpha^2}\right]/\left[1 - \frac{1}{15}a\alpha^2\right] < 1.$$

This is possible, for example, in the case of an air bubble in a liquid when

$\alpha = 0$ и $\eta \gg 1$. At the same time $Rex_{pi} \sim \sqrt{3/\eta}$ a lmx_{pi} 3/2 a η, what does the existence of acutely resonant low-frequency oscillations mean, since

$$(R_e x_{pi}/lmx_{pi}) \gg 1$$

Conclusion

Thus, the oscillations of homogeneous spherical bodies in an infinitely elastic medium were compared. Depending on the various geometric and parametric parameters of the system, a range of complex mechanical systems is determined, located in an elastic environment. When we consider a rigid sphere not in the elastic phase, but in an infinitesimal elastic medium, the equation shows how it manifests itself in partial derivatives

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